Music of the week (Spanish):

Home| Appelquist-Carazzone Decoupling Theorem Anomalous Magnetic Dipole Moment Algebraic Quantum Field Theory A-Theorem 4-fermion Interactions

4-fermion Interactions

Links:

- WIKIPEDIA - Four-fermion Interactions
- WIKIPEDIA - Fermi's Interaction

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Admin • 2 years ago
Hi and welcome to this BLOG.

A-Theorem

The **A-Theorem** states that the **renormalization group (RG)** flux of a 4 dimensional **quantum field theory** is irreversible. That is, if there exist two **conformal field theories** A, B such that one can flow from A to B, one cannot also flow from B to A.

The a-theorem is the analogue of the **c-theorem** in two dimensions.

Papers:

- Holographic C-theorems in Arbitrary Dimensions (2010) - R. C. Myers, A. Sinha local pct. 196
- On Renormalization Group Flows in Four Dimensions (2011) - Z. Komargodski, A. Schwimmer local pct. 176
- Weyl Consistency Conditions and a Local Renormalisation Group Equation for General Renormalisable Field Theories (1991) - H. Osborn local pct. 114

Links:

- Proof Found for Unifying Quantum Principle (2011) - E. S. Reich

Videos:

- Conformal Field Theory - Lecture 5 (2011) - J. Gomis

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Algebraic Quantum Field Theory

> AQFT is our best story about where QFT lives in the mathematical universe, and so is a natural starting point for foundational inquiries.
> - [1] -

Algebraic Quantum Field Theory (**AQFT**) is based on the assumption that the **algebra** of observables represents the core physical structure of **quantum field theory** and on a certain notion of locality. In general the algebra is a ***-algebra**.
No quantum fields appear in the formulation and it can also incorporate extended objects which generalize the

field concept. Thus the more appropriate name **Local Quantum Physics** is also used.

The algebraic framework is very flexible and has proven to be consistent with the developments in elementary particle physics for several decades; besides a formulation of AQFT and **AQFT on curved spacetime**, it can be used to formulate **quantum mechanics** and **Algebraic *Statistical Mechanics*** (**AQSM**).

Historical

AQFT was invented by Rudolf Haag and Daniel Kastler in the 1960s [2], based on the deep insight that the full physical information of a theory is already contained in the net structure, i.e. the respective map from space–time regions to algebras. Phrased differently, equivalent quantum field theories can be identified by the fact that they generate isomorphic local nets.
Daniel Kastler (2003) writes:

> After Rudolf had invited me to spend a year in Urbana, he confronted me with several a priori unrelated insights, one of them based on the postulate that King Solomon could not decide between two physicists working with "physically equivalent representations" of the same C*-algebra. After months of inconclusive investigations of his claims, I had the luck of finding a theorem of Fell in the bibliography of Guichardet’s thesis (which I had providentially taken with me) verifying all of Rudolf's prophecies. The resulting coherence of vision led us to write an article on "An algebraic approach to quantum field theory" which was a hit, perhaps because it seemed to propose a new way of combining physics and mathematics. This paper formulated an axiomatic foundation for the net of local algebras.

See also:

- *Algebraic quantum gravity*

Papers:

- [1] An Algebraic Approach to Quantum Field Theory (1964) - R. Haag, D. Kastler local pct. 982
- Generally Covariant Quantum Field Theory and Scaling Limits (1987) - K. Fredenhagen, R. Haag local pct. 111
- [2] Algebraic Quantum Field Theory (2006) - H. Halvorson local pct. 84
- Some Basic Concepts of Algebraic Quantum Theory (1968) - J. E. Roberts, G. Roepstorff local pct. 41
- Current Trends in Axiomatic Quantum Field Theory (1998) - D. Buchholz local pct. 26
- Algebraic Quantum Field Theory: A Status Report (2000) - D. Buchholz local pct. 21
- Extensions of Automorphisms and Gauge Symmetries (1993) - D. Buchholz, S. Doplicher, R. Longo, J. E. Roberts local pct. 20

Theses:

- An Analysis of the 'Thermal-Time Concept' of Connes and Rovelli (2010) - T.-T. Paetz local

Links:

- WIKIPEDIA - Local Quantum Field Theory
- AQFT in nLab
- Website Hans Halverson

Videos:

- Algebraic Quantum Field Theory - the first 50 Years (2009)

Books:

- Local Quantum Physics: Fields, Particles, Algebras (1996) - R. Haag bct. 2042

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Anomalous Magnetic Dipole Moment

Links:

- WIKIPEDIA - Anomalous Magnetic Dipole Moment

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Appelquist-Carazzone Decoupling Theorem

In **Quantum field theory**, the independence of lower energy phenomena from higher energy ones is formally known as the **Appelquist-Carazzone Decoupling Theorem** [1]. It says that massive fields effectively decouple at low energy: Given a **renormalizable** *Lagrangian* containing both massless and massive fields, one can describe its low energy behaviour with a renormalizable Lagrangian written in terms of the massless fields only. The massive fields only contribute to the low energy Lagrangian through the renormalization of its *couplings* and fields.

More generally, one can eliminate heavy fields from a Lagrangian which also contains light fields by encoding their effects in (generally non-renormalizable) interaction terms involving the light fields only. The resulting *effective field theory* is valid at energies below the masses of the eliminated fields. The whole edifice of standard model extensions - technicolor, **supersymmetry**, **grand unified theories**, **supergravity** and **string theory** with its infinite tower of massive excitations - implicitly depends on the suppression by powers of energy over mass of the effective interaction terms induced by the high energy extensions.

Examples

- One doesn't need detailed knowledge of physics at the scales of grand unification or **quantum gravity** to understand physics at the **electroweak scale**.
- One doesn't need detailed knowledge of physics at the electroweak scale to understand nuclear physics.
- One doesn't need detailed knowledge of nuclear physics to understand chemistry.

Journals:

- Infrared Singularities and Massive Fields (1975) - T. Appelquist, J. Carazzone jct. 1652

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Asymptotic Conditions

In **quantum field theory** the **Asymptotic Conditions** for scalar fields are defined by

$$\lim_{x_0 \to -\infty} \langle\alpha|\hat{\phi}(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\hat{\phi}_{in}(x)|\beta\rangle$$
$$\lim_{x_0 \to \infty} \langle\alpha|\hat{\phi}(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\hat{\phi}_{out}(x)|\beta\rangle$$

where $|\alpha\rangle$ and $|\beta\rangle$ are any pair of *Heisenberg* **states**. $\hat{\phi}_{in}(x)$, $\hat{\phi}_{out}(x)$ are the asymptotically free Heisenberg fields, $\hat{\phi}(x)$ is a Heisenberg field involving interactions. (Strictly speaking, the field operators must be appropriately "smeared out" which is achieved by spatially localized wave packets). Z is a **renormalization constant**.

As the states are elements of a **Hilbert space**, the expressions can be interpreted as (potentially) infinite dimensional matrices. For infinite dimensional spaces, convergence is less trivial. Convergence on the matrix level is called **Weak Convergence**, whereas on the operator level, it is called **Strong Convergence**. Strong convergence implies weak convergence but the converse is not generally true.

It can be shown that if one assumes strong instead of weak convergence, then the **S-matrix** becomes trivial and no scattering takes place.

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Asymptotic Expansion

See also:

- **Divergent series**

Papers:

- Asymptotic Phenomena in Mathematical Physics (1955) - K. O. Friedrichs local pct. 155
- The Devil's Invention: Asymptotic, Superasymptotic and Hyperasymptotic Series (2000) - J. P. Boyd local pct. 108

Links:

- WIKIPEDIA - Asymptotic Expansion

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Bisognano-Wichmann Theorem

In **local relativistic quantum theory** a model is specified in terms of a net of local observable algebras and a **representation** of the **Poincaré group** under which the net is covariant.
The **Bisognano-Wichmann Theorem** intimately connects these two algebraic and geometric aspects.
It asserts that under certain conditions **modular covariance** is satisfied:
The **modular unitary group** of the observable algebra associated to a wedge region coincides with the unitary group representing the boosts which preserve the wedge.
Since the boosts associated to all wedge regions generate the Poincaré group, modular covariance implies that the representation of the Poincaré group is encoded intrinsically in the net of local algebras.

This has further important consequences: It implies the **spin-statistics theorem** and the **CPT theorem**. It also implies
essential *Haag duality* which is an important input to the structural analysis of charge *superselection* sectors.

Counterexamples demonstrate that modular covariance does not follow from the basic principles of **quantum field theory** without further input.
But

- Bisognano and Wichmann have shown modular covariance to hold in theories where the field algebras are generated by finite-component *Wightman fields*.
- In the framework of **algebraic quantum field theory**, Borchers has shown that the modular objects associated to wedges have the correct commutation relations with the translation operators as a consequence of the positive energy requirement.
- Based on Borcher's result, Brunetti, Guido and Longo derived modular covariance for **conformally covariant** theories.

Papers:

- On the Duality Condition for Quantum Fields (1976) - J. J. Bisognano, E. H. Wichmann local pct. 269
- The Double-wedge Algebra for Quantum Fields on Schwarzschild and Minkowski Spacetimes (1985) - B. S. Kay local pct. 88
- The Bisognano-Wichmann Theorem for Massive Theories (2001) - J. Mund local pct. 48

Documents:

- THE BISOGNANO-WICHMAN THEOREM & NETS ON $\mathbb{R}^4$ (2010) - C. Solveen local

Links:

- Bisognano-Wichmann theorem in nLab

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Constructive Quantum Field Theory

> Quantum field theory is a cornerstone of our tentative of interpreting the data obtained by our senses and instruments - the extensions of our senses - that constitute what we call real world.

> Quantum field theory is a tentative to go into some of the inmost folds of these perceptions, a look at scales so small and so far from the daily intuition that we can visualize them in our mind just by constructing a sort of toy models for helping our imagination.
> - Paolo Maria Mariano -

The goal of **Constructive Quantum Field Theory** is to construct interacting models based on the ideas of **renormalization** theory. As yet, success and failure lie close together: It proved possible to construct a whole family of interacting models in two spacetime dimensions such as the $P(\phi)_2$ models, the polynomial models. (Lower indices in this context always mean the spacetime dimension). Two models, ϕ_3^4 and Y_3, the **quartic interaction** and the **Yukawa coupling** were constructed in three spacetime dimensions but, the methods did not lead to any theories in the physical four dimensional spacetime. Instead it is believed that attempts to construct ϕ_4^4 or **quantum electrodynamics** in this way actually lead to free field models.

Implementations

The traditional basis of constructive quantum field theory is the set of *Wightman axioms*. The examples with $d < 4$ satisfy the Wightman axioms as well as the *Osterwalder-Schrader axioms*. They also fall in the related framework of **algebraic quantum field theory** based on the *Haag-Kastler axioms*.

Papers:

- Constructive Quantum Field Theory (2000) - A. Jaffe local pct. 22

Presentations:

- Constructive Quantum Field Theory (2009) - D. Colosi local

Links:

- WIKIPEDIA - Constructive Quantum Field Theory

Videos:

- (Perspectives on nearly 50 Years of) Constructive Quantum Field Theory (2012) - A. Jaffe

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

LSZ Reduction Formula

> The quantum theory of scattering is considerably simplified if we suppose, as was done by Lehmann, Symanzik and Zimmermann, that there is a correspondence between particles and fields. A scheme of this sort, which is a special case of the Wightman approach, is called the LSZ (Lehmann-Symanzik-Zimmermann) formalism. An essential element of it is the notion of chronological products and Green's functions ... The existence of T-products of fields has not been proved from the Wightman axioms in the general case; therefore in the LSZ formalism the existence of T-products is accepted as an additional postulate. The original statements of LSZ (in the modern account) consist of two halves: the Wightman axioms ... and the supplementary requirements.
> - [1] -

The **LSZ Reduction Formula** establishes a relationship between **S-matrix** elements and vacuum expectation values of *time ordered* field operators in the *Heisenberg picture*, i.e. with (off-shell) *Green's functions*.

For a **Klein-Gordon field** the LSZ formula reads:

$$\langle p_1,\dots,p_n;\ \mathrm{out}|q_1,\dots,q_m\ \mathrm{in}\rangle = \langle p_1,\dots,p_n\ \mathrm{in}|S|q_1,\dots,q_m\ \mathrm{in}\rangle =$$

$$\text{"disconnected terms"} + \int \prod_{i=1}^{m}\left\{\mathrm{d}^4 x_i\, \frac{ie^{-iq_i\cdot x_i}\left(\Box_{x_i}+m^2\right)}{Z^{\frac{1}{2}}}\right\}\prod_{j=1}^{n}\left\{\mathrm{d}^4 y_j\, \frac{ie^{ip_j\cdot y_j}\left(\Box_{y_j}+m^2\right)}{Z^{\frac{1}{2}}}\right\}\langle\Omega|\mathcal{T}\phi(x_1)\dots\phi(x_m)\phi(y_1)\dots\phi(y_n)|\Omega\rangle$$

Derivation

Since no recourse to **perturbation theory** is required in the following, we do not need to make any reference to free Hamiltonians, bare vacua, **interaction picture** fields, etc. All fields will be in the Heisenberg representation and **states** will refer to eigenstates of the full Hamiltonian. (The vacuum $|\Omega\rangle$ therefore is to be understood as the vacuum of the full theory not to be confused with the vacuum $|0\rangle$ of a non-interacting theory. In fact these can be related, using the *Gell-Mann and Low formula*).

We start by creating a particle with momentum q_1 from the in-state according to

$$\langle f|i\rangle = \langle p_1,\ldots,p_n;\mathrm{out}|q_1,\ldots,q_m;\mathrm{in}\rangle = \langle p_1,\ldots,p_n;\mathrm{out}|a^\dagger_{in}(q_1)|q_2,\ldots,q_m;\mathrm{in}\rangle$$

The *creation operator* $a^\dagger(q_1)$ can be written as

$$a^\dagger_{in}(p_1) = \frac{1}{i}\int d^3x\; e^{-iq_1x}\overleftrightarrow{\partial}_0\phi_{in}(x)$$

Hence

$$\langle f|i\rangle = \frac{1}{i}\int d^3x\; e^{-iq_1x}\overleftrightarrow{\partial}_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi_{in}(x)|q_2,\ldots,q_m;\mathrm{in}\rangle$$

We use the **relation** between the in-fields and the interacting fields,

$$\langle\alpha|\phi_{in}(x)|\beta\rangle = \frac{1}{\sqrt{Z}}\lim_{t\to-\infty}\langle\alpha|\phi(x)|\beta\rangle$$

leading to

$$\langle f|i\rangle = \frac{1}{i\sqrt{Z}}\lim_{t\to-\infty}\int d^3x\; e^{-iq_1x}\overleftrightarrow{\partial}_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle$$

Let's define

$$f(t) = \frac{1}{i\sqrt{Z}}\int d^3x\; e^{-iq_1x}\overleftrightarrow{\partial}_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle$$

and use the mathematical identity

$$\lim_{t\to\infty}f(t) - \lim_{t\to-\infty}f(t) = \lim_{\substack{t_i\to-\infty\\ t_f\to\infty}}\int_{t_i}^{t_f}\frac{\partial f(t)}{\partial t}\;dt$$

where $f(t)$ is some arbitrary function, which allows us to connect the in- with the out-states.
We get

$$\begin{aligned}\langle f|i\rangle &= \lim_{t\to\infty}f(t) - \lim_{\substack{t_i\to-\infty\\ t_f\to\infty}}\int_{t_i}^{t_f}\frac{\partial f(t)}{\partial t}\;dt\\ &= \frac{1}{i\sqrt{Z}}\lim_{t\to\infty}\int d^3x\; e^{-iq_1x}\overleftrightarrow{\partial}_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle\\ &\quad-\frac{1}{i\sqrt{Z}}\lim_{\substack{t_i\to-\infty\\ t_f\to\infty}}\int d^4x\;\partial_0\left(e^{-iq_1x}\overleftrightarrow{\partial}_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle\right)\end{aligned}$$

Using

$$\lim_{t\to\infty}\langle\alpha|\phi(x)|\beta\rangle = \sqrt{Z}\langle\alpha|\phi_{out}(x)|\beta\rangle$$

the first terms can be rewritten as

$$\langle p_1,\ldots,p_n;\mathrm{out}|a^\dagger_{out}(q_1)|q_2,\ldots,q_m;\mathrm{in}\rangle$$

where $a^\dagger_{out}$ now acts to the left. If an out-state with momentum $p_i = q_1$ exists, it is annihilated, otherwise the term is zero. Physically such a term, if nontrivial, can be understood as a particle that doesn't participate in the interaction process. It is also called **"Disconnected" Term**. These terms are not really interesting in scattering theory. For this reason one also uses the T-matrix $iT \equiv S - 1$ instead, where the disconnected contributions are omitted.
Consequently the nontrivial part concerning the interactions must be contained in the second term. Carrying out the innermost derivative with respect to time, it becomes

$$-\frac{1}{i\sqrt{Z}}\lim_{\substack{t_i\to-\infty\\ t_f\to\infty}}\int d^4x\;\partial_0\left(e^{-iq_1x}\partial_0\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle - (-ip_{1_0})e^{-iq_1x}\langle p_1,\ldots,p_n;\mathrm{out}|\phi(x)|q_2,\ldots,q_m;\mathrm{in}\rangle\right)$$

Carrying out the second derivative with respect to x_0 leads to four terms, two of which cancel, leaving us with

$$- \frac{1}{i\sqrt{Z}} \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int d^4x \, \left(e^{-iq_1 x} \partial_0^2 \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle + (p_{1_0})^2 e^{-iq_1 x} \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle \right)$$

Plugging in the energy momentum relation $q_{1_0}^2 - \vec{q}_1^{\,2} = m^2$ and using

$$\vec{\nabla}^2 e^{-iq_1 x} = -\vec{q}_1^{\,2} e^{-iq_1 x}$$

leads to

$$- \frac{1}{i\sqrt{Z}} \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int d^4x \, \left(e^{-iq_1 x} (\partial_0^2 + m^2) \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle - (\vec{\nabla}^2 e^{-iq_1 x}) \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle \right)$$

Doing a spatial integration by parts twice results in two surface terms and one volume term. We neglect the surface terms in the following. This point is somewhat subtle, but it is at least justified by the great success of the LSZ formalism in practice. On theoretical grounds, one has to keep in mind that we have used the asymptotic conditions above which are based on fields that are smeared out, that is, the $\phi(x)$ can be regarded as (spatial) wave packets. It is reasonable to assume that these are localized on the scale of the experimental setup. (E.g. if we take infinity to be the radius of the orbit of Pluto, the probability of finding a particle there is completely negligible compared to the scattering probabilities of interest in our experiment).
Therefore the second term becomes

$$\begin{aligned} & - \frac{1}{i\sqrt{Z}} \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int d^4x \, \left(e^{-iq_1 x} (\partial_0^2 + m^2) \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle - e^{-iq_1 x} \vec{\nabla}^2 \langle p_1, \ldots, p_n; \text{out}|\phi(x)|q_2, \ldots, q_m; \text{in}\rangle \right) \\ = \; & - \frac{1}{i\sqrt{Z}} \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int d^4x_1 \, e^{-iq_1 x_1} (\Box_{x_1} + m^2) \langle p_1, \ldots, p_n; \text{out}|\phi(x_1)|q_2, \ldots, q_m; \text{in}\rangle \end{aligned}$$

Next, we consider the creation of a particle with momentum q_1 from the out-state. Proceeding in close analogy to previous steps, we get

$$\begin{aligned} & \langle p_1, \ldots, p_n; \text{out}|\phi(x_1)|q_2, \ldots, q_m; \text{in}\rangle = \\ & \langle p_2, \ldots, p_n; \text{out}|a_{out}(p_1)\phi(x_1)|q_2, \ldots, q_m; \text{in}\rangle = \\ & i \int d^3y \, \langle p_2, \ldots, p_n; \text{out}| \left(e^{ip_1 y} \overleftrightarrow{\partial_0} \phi_{out}(y) \right) \phi(x_1)|q_2, \ldots, q_m; \text{in}\rangle = \\ & \lim_{t \to \infty} \frac{i}{\sqrt{Z}} \int d^3y \, \langle p_2, \ldots, p_n; \text{out}| \left(e^{ip_1 y} \overleftrightarrow{\partial_0} \phi(y) \right) \phi(x)|q_2, \ldots, q_m; \text{in}\rangle \end{aligned}$$

In the next step the analogy breaks down because we cannot let $\phi(y)$ act to the right on the in-state as there is another field operator in between. We therefore must somehow make it to flip the two operators. I.e. what we want is something of the kind

$$\lim_{t \to \infty} f[\phi(y), \phi(x)] = \lim_{t \to -\infty} f[\phi(x), \phi(y)] + \text{something}$$

The trick is to introduce time ordering. Actually the terms are already time ordered, so we can equivalently write

$$\lim_{t \to \infty} \mathcal{T} f[\phi(y), \phi(x)] = \lim_{t \to -\infty} \mathcal{T} f[\phi(x), \phi(y)] + \text{something}$$

We now use the mathematical identity from above, but this time with $\mathcal{T} f$ instead f

$$\lim_{t \to \infty} \mathcal{T} f(t) - \lim_{t \to -\infty} \mathcal{T} f(t) = \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int_{t_i}^{t_f} \frac{\partial \mathcal{T} f(t)}{\partial t} \, dt$$

I.e. the price we have to pay for flipping the operators is the time ordering in the (nontrivial) "compensating" interaction term. It is worth understanding this point well, because this is one of the origins of the omnipresent time ordering in quantum field theory.
The second term again is disconnected and thus we'll not further dwell on it.
The interaction term reads

$$- \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \frac{i}{\sqrt{Z}} \int d^4y \, \langle p_2, \ldots, p_n; \text{out}| \mathcal{T} \left(\partial_0 \left(e^{ip_1 y} \overleftrightarrow{\partial_0} \phi(y) \right) \right) \phi(x)|q_2, \ldots, q_m; \text{in}\rangle$$

It can be resolved in a similar fashion as before. Doing this and putting everything together leads to

$$= \left(\frac{1}{i\sqrt{Z}} \right)^2 \lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} \int d^4y_1 \int d^4x_1 \, e^{-iq_1 x_1} e^{ip_1 y_1} (\Box_{x_1} + m^2)(\Box_{y_1} + m^2) \langle p_2, \ldots, p_n; \text{out}| \mathcal{T} \phi(x_1)\phi(y_1)|q_2, \ldots, q_m; \text{in}\rangle$$

We have flipped the order of the fields which is allowed as they are time ordered.

The two kind of steps can be interatively repeated until all states are removed from the in an out states, leaving

us with the in- and out vacua which we assume to be the same, namely the $|\Omega\rangle$ described above.
The resulting formula is the one to be derived.

Applications

Besides its usefuleness for calculating scattering amplitudes, in *statistical physics*, the LSZ formalism allows for obtaining a general formulation of the *fluctuation-dissipation theorem*.

Limitations

The LSZ reduction formula in its original form cannot treat **bound states**, massless particles and topological **solitons** (a.k.a. topological defects). For example, in **QCD**, the asymptotic states are bound (a phenomenon known as **confinement**), i.e. they are not free states as required by the LSZ formalism.
However the formula can at least be generalized in order to include bound states, which are states described by non-local composite fields.

Links:

- WIKIPEDIA - LSZ Reduction Formula
- WIKIPEDIA - Källén-Lehmann Spectral Representation

Lectures:

- Quantum Field Theory I (2011) - M. Luke local

Videos:

- Interacting Field Theory - VI - P. Tripathy VII
- Einführung in die Quantenfeldtheorie und das Standardmodell der Teilchenphysik (2013) - M. Wagner

Books:

- [1] General Principles of Quantum Field Theory (1990) - N. N. Bogolubov, A. A. Logunov, A. I. Oksak, I. Todoro bct. 568 - Chapter 9

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Nonassociative Quantum Field Theory

Papers:

- One-loop Unitarity of Scalar Field Theories on Poincaré Invariant Commutative Nonassociative Spacetimes (2006) - Y. Sasai, N. Sasakura local pct. 19
- Nonperturbative Operator Quantization of Strongly Nonlinear Fields (2002) - V. Dzhunushaliev local pct. 10
- Particle Scattering in Nonassociative Quantum Field Theory (1996) - V. D. Dzhunushaliev local pct. 4

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Noncommutative Quantum Field Theory

Noncommutative Quantum Field Theory (**NQFT**) is a **quantum field theory** based on a **noncommutative spacetime**.

Papers:

- Noncommutative Field Theory and Lorentz Violation (2001) - S. M. Carroll, J. A. Harvey, V. A. Kostelecký, C. D. Lane, T. Okamoto local pct. 522
- General Properties of Noncommutative Field Theories (2003) - L. Álvarez-Gaumé M. Á. Vázquez-Mozo local pct. 159
- Vanishing of Beta Function of Non Commutative $\Phi^4{}_4$ Theory to all Orders (2006) - M. Disertori, R. Gurau, J. Magnen, V. Ivasseau local pct. 96
- The β-function in Duality-covariant Noncommutative ϕ^4-theory (2004) - H. Grosse, R. Wulkenhaar local pct. 62

Lectures:

- Introduction to Noncommutative QFT (2010) - M. Wohlgenannt local

Links:

- WIKIPEDIA - Noncommutative Quantum Field Theory

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

One-loop Effective Action

Given a bare action $S[\phi]$, the **One-loop** ***Effective Action*** is

$$\Gamma^{(1)}[\phi] = \frac{\varepsilon}{2}\frac{\hbar}{i}\,\mathrm{Tr}\ln\left(\frac{\delta^2 S[\phi]}{\delta\phi\delta\phi}\right)$$

where ϕ is an arbitrary set of background fields (mean fields), $\varepsilon = +1$ for bosonic fields and $\varepsilon = -1$ for fermionic fields.

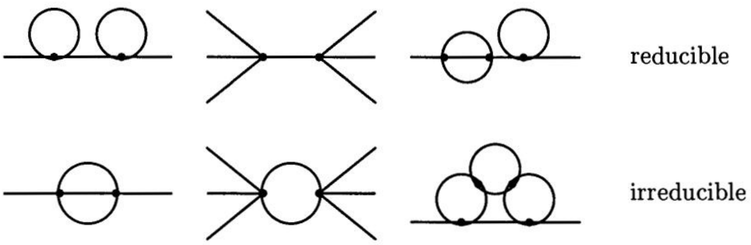
In many cases, the one-loop contribution to the effective action contains the most relevant information of the quantum effects in the low energy regime.

Links:

- WIKIPEDIA - One-loop Feynman Diagram

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

One-particle Irreducible Diagram



A **One-particle Irreducible Diagram** (short **1PI Diagram**) is a *Feynman diagram* which cannot be divided into two diagrams by removing a single internal line (i.e. a *propagator*).

The irreducible diagrams play an important role for the systematic construction of **perturbation theory** in higher orders. Every higher-order diagram can be obtained in a unique way by taking irreducible diagrams and free *propagators* as building blocks.

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Philosophical Aspects of Quantum Field Theory

> K.O. Friedrichs described his feelings about the literature on quantum field theory as akin to the challenge felt by an archeologist stumbling on records of a high civilization written in strange symbols. Clearly there were intelligent messages, but what did they want to say?
> - Rudolf Haag -

The concept of a quantized field to describe particles is that there is an infinite field of potential particles. The ground state of this field is the **vacuum**. This is a state where there are no particles but in the quantum world just like the harmonic oscillator this does not correspond to zero energy. The vacuum must therefore be though of not as empty but as being full of potential particles. There is also not only one field to be considered. The scalar field gives a description of scalar particles but for fermions a fermionic field is necessary. This is not

the end as every different type of particle must have its own field. The vacuum then becomes the ground state of
every possible type of particle at every possible spacetime point. When a particle is created it is like one of these potential particles is pulled form the ground state. When a particle is annihilated it is put back into the potential sea. This explains how in an interaction any imaginable particle can be created at any point in spacetime. This also explains why particles of the same type are identical. This seems like a triviality but there is no reason why particles created at huge intervals in space and time should be identical unless they come from the same source.

See also:

- *What is a quantum field ?*

Papers:

- What is Quantum Field Theory, and What Did We Think It Is? (1997) - S. Weinberg local pct. 217 - "In its mature form, the idea of quantum field theory is that quantum fields are the basic ingredients of the universe, and particles are just bundles of energy and momentum of the fields. In a relativistic theory the wave function is a functional of these fields, not a function of particle coordinates. Quantum field theory hence led to a more unified view of nature than the old dualistic interpretation in terms of both fields and particles."
- No Place for Particles in Relativistic Quantum Theories? (2001) - H. Halvorson, R. Clifton local pct. 101
- In Defense of Dogma: Why there cannot be a Relativistic Quantum Mechanics of (Localizable) Particles (1996) - D. B. Malament local pct. 78
- Quantum Field Theory (1999) - F. Wilczek local pct. 66 - "What are the essential features of quantum field theory?...The ...question has no sharp answer. Theoretical physicists are very flexible in adapting their tools, and no axiomization can keep up with them. However I think it is fair to say that the characteristic, core ideas of quantum field theory are twofold. First, that the basic dynamical degrees of freedom are operator functions of space and time - quantum fields, obeying appropriate commutation relations. Second, that the interactions of these fields are local. Thus the equations of motion and commutation relations governing the evolution of a given quantum field at a given point in space-time should depend only on the behavior of fields and their derivatives at that point."
- The Quest for Understanding in Relativistic Quantum Physics (1999) - D. Buchholz, R. Haag local pct. 60
- Are Rindler Quanta Real? Inequivalent Particle Concepts in Quantum Field Theory (2000) - R. Clifton, H. Halvorson local pct. 53
- Against Field Interpretations of Quantum Field Theory (2009) - D. J. Baker local pct. 26
- Quantum Field Theory: Underdetermination, Inconsistency, and Idealization (2009) - D. Fraser local pct. 20
- Rough Guide to Spontaneous Symmetry Breaking (2003) - J. Eearman local pct. 15
- Relativistic Quantum Mechanics and Field Theory (2004) - F. Strocchi local pct. 14
- What is Quantum Field Theory and why have some Physicists Abandoned it? (1997) - R. Jackiw local pct. 6
- Not Particles, Not Quite Fields: An Ontology for Quantum Field Theory - T. Lupher local pct. 1

Links:

- How To Think About Quantum Field Theory (2012) - S. Carroll
- Stanford Encyclopedia of Philosophy - Quantum Theory: von Neumann vs. Dirac
- Website of David John Baker
- John Earman Bibliography

Videos:

- Philosophy of Quantum Field Theory Conference (2009)
- Taking Particle Physics Seriously: A Critique of the Algebraic Approach to Quantum Field Theory (2009) - D. Wallace

Books

- Ontological Aspects of Quantum Field Theory (2002) - M. Kuhlmann, H. Lyre, A. Wayne bct. 40

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Quantum Electrodynamics

Papers:

- The Radiation Theories of Tomonaga, Schwinger, and Feynman (1949) - F. J. Dyson local pct. 1008

Lectures:

- Feynman Rules for QED local

Videos:

- Feynman Rules in QED II - P. Tripathy
- Lecture on Quantum Electrodynamics: QED (1979) - R. Feynman
- Linking the Ideas of Feynman, Schwinger and Tomanaga (1998) - F. Dyson

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Quantum Field Cellular Automaton

The idea is that any **tunneling** between two **unitarily inequivalent vacua** of a **quantum field theory** defines an elementary "time step" of a **cellular automaton**, which in this case will be called a **Quantum Field Cellular Automaton** (**QFCA**).

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Quantum Field Computer

It's hard to have an idea and somebody didn't have it before. This is what happened to me with the **Quantum Field Computer** (**QFC**), although it seems I have a bit of a different take on the subject. More details can be found under **unitary inequivalence**.
Quantum field computing, which is computing with the "continuum" rather than digital computing, I suspect to be the ultimate computing paradigm, way superior to any form of classical or **quantum computing**. I think it should allow to proof the **Church-Turing Hypothesis** to be wrong. In its full fledged form it must involve all the forces of nature, in particular **gravity**. (An implementation though requires a better understanding of **quantum gravity**). Moreover it should allow for answering the question as to how to construct real **AI** and solve the **"hard problem" of consciousness** in philosophy, paving the way to building "conscious machines". If the conscious mind involves computations based on **quantum field theory**, there had to be elements of **non-computability**, something which has already been suspected by some people (e.g. **Roger Penrose**). For a good understanding of the subject, it would be crucial to know how to build a quantum field computer "from scratch". At the moment I do not have a good idea how to do this.

If the human **brain** is a quantum field computer, then it must have an uncountably infinite number of states. If one divides this number by the number of atoms in the brain (which is finite), one is left with an uncountably infinite number of states. The consequence is that elementary particles must already be conscious and are the building blocks of "higher" forms of consciousness. Again, not really a new idea. Thus an elementary particle is a small cosmos all by its own, having an incredible computational capacity, by far exceeding that of any classical computer. Therefore, emulating such a system on a classical computer will never be possible. The best one can do is to approximate it to a certain degree.

Some postulates

A QFC ...

- is non-**deterministic**. Although it may be constrained to a certain degree, leading to *superselection rules*. Thus in a way any quantum field computer has generic **free will**.

- can emulate biological systems (**quantum field biology**).

- can simulate a **Hilbert hotel**. A rearrangement in the hotel can be interpreted as an elementary operation of a QFC. (An example would be a "global shift operation", letting all people go to rooms with even numbers). The point is that any operation is global, i.e. it does not involve the propagation of a signal limited by the speed of light. An idea is that this "update" of Hilbert's hotel is a *quantum tunneling* process between inequivalent quantum vacua. I.e. the paradigm of a QFC would be very much that of an **infinite cellular automaton** with Hilbert's hotel being a nice illustrative example. The crucial difference between a Turing machine and a QFC is that the former only can do local changes (on the Turing strip), i.e. a finite number of bits are flipped at a time, whereas the latter is bound to do an infinite number of changes of states per time step otherwise there is no transition to a new vacuum.

- never "crashes" - it just can't do so by definition. Concerning nature, what has been very intriguing to me is that if it is doing a huge computation (some even believe it's a **simulation**) based on a "digital" algorithm, why does it "never" crash ("never", for all practical purposes) ? But if the fundamental building blocks of nature are quantum fields - which is state of the art of our understanding - then an explanation is at hand. (See also [1] for more details).

Questions

- What is the smallest QFC possible ? The answer could come from biology and "living" systems.

- Can we find a generic QFT effect we "cannot put on a conventional machine". (There are some hints of such effects, e.g. *chiral fermions* in **lattice field theory**).

... etc. pp. - an awful lot remains to be understood !

See also:

- **Quantum field cellular automaton**
- **Quantum field biology**
- **Digital physics**
- **Is nature infinite ?**

Papers:

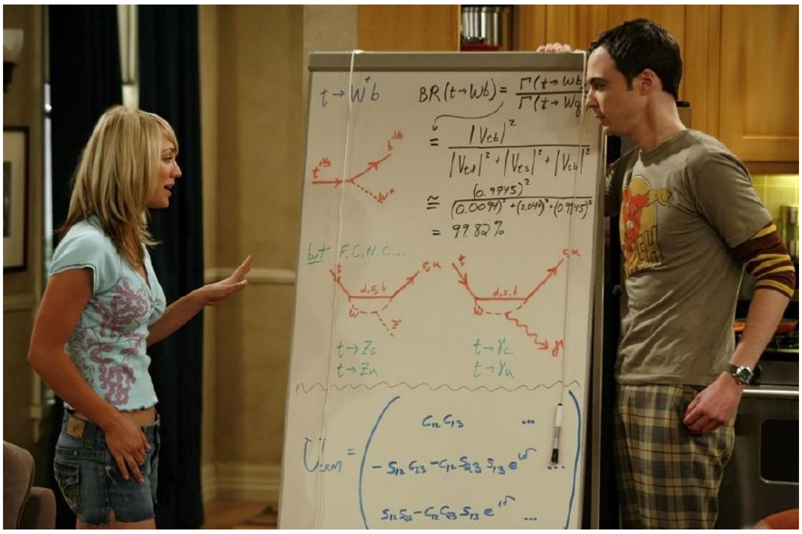
- P/NP, and the Quantum Field Computer (1998) - M. H. Freedman local pct. 115
- Quantum Algorithms for Quantum Field Theories (2011) - S. P. Jordan, K. S. M. Lee, J. Preskill local pct. 49
- [1] The Dissipative Brain (2004) - G. Vitiello local pct. 36
- The Unity between Quantum Field Computation, Real Computation, and Quantum Computation (2001) - A. C. Manoharan local pct. 3
- Beyond Quantum Computation and Towards Quantum Field Computation (2003) - A. C. Manoharan local pct. 0
- QFT + NP = P Quantum Field Theory (QFT): A Possible Way of Solving NP-Complete Problems in Polynomial Time (1996) - A. Beltran, V. Kreinovich, L. Longpré local pct. 0

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Quantum Field Theory

> The breakthrough in the handling of Quantum Electrodynamics ... had restored faith in the power of quantum fiel theory. But side by side with the dominant feeling of great triumph there was a spectrum of mixed feelings ranging from bewilderment to severe criticism.
> Dirac emphasized that there was no acceptable physical theory but only an ugly set of rules.
> Heisenberg felt that the success of renormalization had turned the minds away from the really important issues in shaping a new theory.
> - Detlev Buchholz, Rudolf Haag [1] -

> There's a saying at Harvard that you don't really understand quantum field theory until you have taken it three times.
> - A. Ananthaswamy [2] -



In essence, **Quantum Field Theory** (**QFT**) is **quantum mechanics** with an **infinite number of degrees of freedom**. Quantum field theory is drastically different from quantum mechanics as in general the **Stone-Von Neumann theorem** does not hold.

QFT is not confined to the relativistic domain, it also applies to non-relativistic many-body systems in **condensed matter** physics.
In this last case, one considers the so-called *thermodynamic limit* in which the infinite volume limit is understood in such a way that the density N/V is kept constant, with N denoting the particle number.
QFT in connection with the quantum many-body problem arises in the theory of metals, **superconductivity**, the low-temperature behavior of the quantum liquids He^3 and He^4, and the **quantum Hall effect**, among others.

While a large number of quantum field theories can be constructed that seem to be free of internal inconsistencies, the rules of the game are rather limited due to such constraints as **anomaly** cancellation, **renormalizability**, and **ghostfreeness**.

See also:

- **Algebraic quantum field theory**
- **Constructive quantum field theory**
- **Thermal quantum field theory**
- **Topological quantum field theory**
- *Lattice quantum field theory*
- **Philosophical aspects of quantum field theory**
- **Quantum field computer**
- **Quantum field biology**
- *What is a quantum field ?*
- *Why quantum field theory?*
- **Nonassociative quantum field theory**

Papers:

- [1] The Quest for Understanding in Relativistic Quantum Physics local pct. 60
- THE UNREASONABLE EFFECTIVENESS OF QUANTUM FIELD THEORY (1996) - R. Jackiw local pct. 21

Links:

- [2] Taming Infinity (2009)- A. Ananthaswamy local

Lectures:

- THE CONCEPTUAL BASIS OF QUANTUM FIELD THEORY (2010) - G. 't Hooft local lct. 3
- Notes from Sidney Coleman's Physics 253a (1986) local lct. 1
- Quantum Field Theory I (2007/8), II - M. G. Schmidt local I II - Heard his lectures in the 90s. (But it was

the first time I took QFT - see quote above. So I had to do another round before making the exam).

Videos:

- Mathematical Foundations of Quantum Field Theory (2012)
- Mathematical Aspects of Quantum Field Theory and Quantum Statistical Mechanics (2012)
- Three Roles of Quantum Field Theory (2011) - G. Segal
- Lectures:
  - David Tong: Lectures on Quantum Field Theory (2009)
  - Quantum Field Theory II (2009) - F. David

Google books:

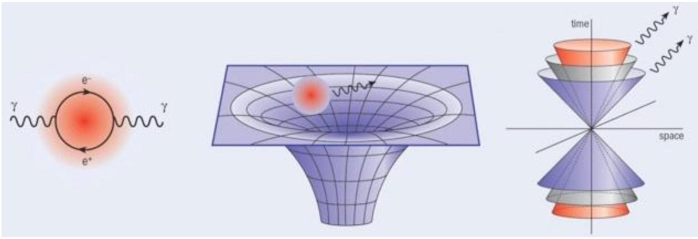
- Mathematical Physics in Mathematics and Physics: Quantum and Operator Algebraic Aspects (2001) - R. Longo bct. 4

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Quantum Field Theory in Curved Spacetime

The subject of **Quantum Field Theory in Curved Spacetime** is the study of the behavior of quantum fields propagating in a classical **gravitational field**. It is used to analyze phenomena where the quantum nature of fields and the effects of gravitation are both important, but where the **quantum nature of gravity** itself is assumed not to play a crucial role, so that gravitation can be described by a classical, curved spacetime, as in the framework of **general relativity**. Its two applications of greatest interest are to phenomena occurring in the very early universe and to phenomena occurring in the vicinity of **black holes**. Despite its classical treatment of gravity, quantum field theory in curved spacetime has provided some of the deepest insights into the nature of quantum gravity so far (e.g. the **Hawking effect**).

Contrary to the standard treatments of quantum field theory in flat spacetime which rely heavily on **Poincaré symmetry** (usually entering the analysis implicitly via plane-wave expansions) and the interpretation of the theory primarily in terms of a notion of "particles", neither Poincaré (or other) symmetry nor a useful notion of "particles" exists in a general, curved spacetime, so a number of the familiar tools and concepts of field theory must be "unlearned" in order to have a clear grasp of quantum field theory in curved spacetime.



One of the technical problems one is facing when doing quantum field theory in a curved background is that there exist **unitarily inequivalent Hilbert space** constructions of free quantum fields in spacetimes with a noncompact *Cauchy surface* and (in the absence of symmetries of the spacetime) none appears "preferred". That is, there is no "preferred" choice of a **vacuum state** and an unambiguous notion of "particles" doesn't exist.
For a free field in Minkowski spacetime, the notion of "particles" and "vacuum" is intimately tied to the notion of "positive frequency solutions", which, in turn relies on the existence of a time translation symmetry. These notions of a (unique) "vacuum state" and "particles" can be straightforwardly generalized to (globally) stationary curved spacetimes, but not to general curved spacetimes. For a free field on a general curved spacetime, one has the general notion of a quasi-free Hadamard state (i.e., vacuum) and a corresponding notion of "particles". However, these notions are highly non-unique - and indeed, for spacetimes with a non-compact Cauchy surface different choices of quasi-free Hadamard states give rise, in general, to unitarily inequivalent Hilbert space constructions of the theory.

The difficulties that arise from the existence of unitarily inequivalent Hilbert space constructions of quantum field theory in curved spacetime can be overcome by formulating the theory via the **algebraic framework**, where the relevant physics is encoded by the algebra of local field observables and where one does not have to specify a choice of state (or representation) to formulate the theory. The algebraic approach also fits in very well with the viewpoint naturally arising in quantum field theory in curved spacetime that the fundamental observables in QFT are the local quantum fields themselves.

For linear fields in curved spacetime, a fully satisfactory, mathematically rigorous theory can be constructed.

See also:

- *Klein-Gordon Field in curved spacetime*

Papers:

- Quantum Field Theory in Curved Spacetime (1975) - B. S. DeWitt local pct. 1134
- On Quantum Field Theory in Gravitational Background (1984) - R. Haag, H. Narnhofer, U. Stein local pct. 154
- Quantization of Scalar Fields in Curved Background and Quantum Algebras (2001) - A. Iorio, G. Lambiase, G. Vitiello local pct. 16
- Quantum Fields in Nonstatic Background: A Histories Perspective (1999) - C. Anastopoulos local pct. 13
- Quantum Field Theory on Curved Backgrounds (2013) -- A Primer M. Benini, C. Dappiaggi, T.-P. Hack local pct. 13

Presentations:

- Quantum Field Theory on Curved Spacetime - Y. Ahmed local

Links:

- WIKIPEDIA - Quantum Field Theory in Curved Spacetime

Videos:

- Axiomatic Quantum Field Theory in Curved Spacetime (2009) - R. M. Wald transparencies local
- The Locally Covariant Approach to Quantum Field Theory in Curved Spacetimes (2008) - C. J. Fewster
- Quantum Field Theory in Curved Spacetime (2007) - R. Wald

Books:

- Quantum Fields in Curved Space (1986) - N. D. Birrell, P. C. W. Davies bct. 6105
- Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (1994) - R. M. Wald bct. 1259 - My favourite book in the subject.

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Reeh-Schlieder Theorem

> Thus, in principle, in a laboratory on earth one can, by artfully manipulating vacuum fluctuations, construct a house on the backside of the moon.
> - Stephen J. Summers -

The **Reeh-Schlieder Theorem** of relativistic **quantum field theory** states that if there where no restrictions on the available energy, one could prepare any vector **state** with arbitrary accuracy using only strictly local operations; i.e., operations performed in an arbitray bounded space-time region. (In the mathematical description, the **vacuum** state is specified by a vector in the so-called vacuum **Hilbert space** H and a local operation is modelled by a linear operator acing on H).
Yet, according to the cluster theorem these correlations decay exponentially (as long as the theory describes only massive particles). Therefore the energy necessary to exploit them puts severe limits on the size of affordable effects.

As a consequence, the vacuum for general quantum fields violates **Bell inequality** and has **entanglement** across causally disconnected regions.

Localisation of more and more energy ultimately leads to **black hole** formation, giving a hint that in a theory of **quantum gravity** the Reeh-Schlieder theorem needs amendments. (E.g. the creation of an **electron** at moon distance already would require so much energy that the laboratory on earth would turn into a black hole).

A consequence of the Reeh-Schlieder theorem is that relativistic quantum field theories do not admit systems of local **number operators**.

* Detection of Vacuum Entanglement in a Linear Ion Trap (2004) - A. Retzker, J. I. Cirac, B. Reznik local pct. 78

- Entanglement and Open Systems in Algebraic Quantum Field Theory (2000) - R. Clifton, H. Halvorson local pct. 61
- Generic Bell Correlation Between Arbitrary Local Algebras in Quantum Field Theory (2000) - H. Halvorson, R. Clifton local pct. 61
- Microlocal Analysis of Quantum Fields on Curved Spacetimes: Analytic Wavefront Sets and Reeh-Schlieder Theorems (2002) - A. Strohmaier, R. Verch, M. Wollenberg local pct. 47
- Dark Energy from Vacuum Entanglement (2007) - J.-W. Lee, J. Lee, H.-C. Kim local pct. 36 TRD
- The Reeh-Schlieder Property for Thermal Field Theories (2004) - C. D. Jäkel local pct. 16
- On the Reeh-Schlieder Property in Curved Spacetime (2008) - K. Sanders local pct. 15

- Separability for Lattice Systems at High Temperature (2004) - H. Narnhofer local pct. 10
- The Reeh-Schlieder Property for Ground States (2000) - C. D. Jäkel local pct. 1

Links:

- WIKIPEDIA - Reeh-Schlieder Theorem

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Regularization

There exist infinitely many different analytic **Regularization** procedures. Therefore the question is: which of the regularizations that are being used is the one chosen by nature? In practice, one always tries to avoid answering this question, by checking the finite results obtained with different regularizations and by comparing them with classical limits which provide well-known, physically meaningful values.

Examples of regularization schemata are:

- **Dimensional regularization**
- *Pauli-Villars regularization*
- *Zeta function regularization*
- *Heat kernel regularization*
- Lattice regularization
- Point splitting regularization

Links:

- WIKIPEDIA - Regularization (Physics)

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Schwinger Proper Time Representation

The **(Fock-)Schwinger Proper-time Representation** (a.k.a. **deWitt-Schwinger Proper-time Representation**) is an integral representation of the logarithm of an operator $\hat{H}$, given by

$$\ln(\hat{H}) = -\int_0^\infty \frac{d\tau}{\tau}\, e^{-\hat{H}\tau}$$

where τ is known as **Proper Time** or **Schwinger Parameter**.
It can also be expressed in terms of $\hat{H}^{-1}$, the *propagator* of $\hat{H}$, for variation with respect to $\hat{H}$ yields

$$\frac{\delta\hat{H}}{\hat{H}} = -\int_0^\infty \frac{d\tau}{\tau}\,(-\tau\delta\hat{H})e^{-\hat{H}\tau}$$

hence

$$\hat{H}^{-1} = \int_0^\infty d\tau e^{-\hat{H}\tau}$$

It follows that

$$\mathrm{Tr}(\ln(\hat{H})) = -\int_0^\infty \frac{d\tau}{\tau}\,\mathrm{Tr}\left(e^{-\hat{H}\tau}\right)$$

which is useful for calculating the **one loop effective action**.
The Schwinger proper-time representation establishes a relationship between the one loop effective action and the **heat kernel** which allows for expressing it in terms of a **heat kernel expansion**.

In the following we give heuristic arguments how one can arrive at the expression for the proper time representation, based on the following identity for the **exponential integral**

$$\mathrm{E}_1(x) = \int_x^\infty \frac{e^{-t}}{t}\,dt = -\ln(x) - \gamma - \sum_{k=1}^{\infty}\frac{(-x)^k}{k\,k!}$$

As the argument of a logarithm is dimensionless, if one wants to introduce an operator, one has to multiply it with a scale.
E.g. $\mathbf{D} \equiv \mathbf{\Delta} x \, \frac{\partial}{\partial x}$ for a **Dirac type operator** or $\boldsymbol{D}^2 \equiv \left(\mathbf{\Delta} x \, \frac{\partial}{\partial x}\right)^2$ for a **Laplace type operator**.
For simplicity, we consider a linear and scalar operator. Moreover we regard the Euclidean case. Then

$$\begin{aligned} E_1(\Delta x \partial_x) &= \int_{\Delta x \partial_x}^{\infty} \frac{d\tau}{\tau} \, e^{-\tau} = -\ln(\Delta x \partial_x) - \gamma - \sum_{k=1}^{\infty} \frac{(-\Delta x \partial_x)^k}{k \, k!} \\ &= \int_{\Delta x}^{\infty} \frac{d\tau}{\tau} \, e^{-\tau \partial_x} \end{aligned}$$

Taking the (UV) limit $\Delta x \to dx \approx 0$ and assuming that the logarithmic term is the dominant one leads to

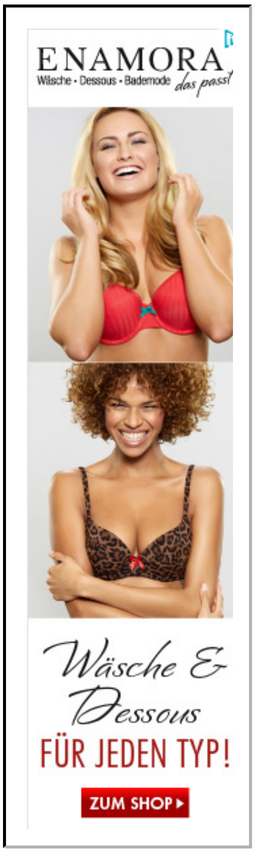
$$\ln(dx \partial_x) = \int_0^{\infty} \frac{d\tau}{\tau} \, e^{-\tau \partial_x}$$

which is nearly the formula above.
There is one subtle difference, namely that in case of the argument of the logarithm one actually has to use the **total differential** associated with the operator, which in the finite case amounts to multiplying the operator with a scale to make it dimensionless.
The expression above (often found in literature) therefore has to be seen as formal with an implicit assumption.
At any rate, the **regularized** version of the integral is more realistic anyway. Using a regularization scale (cutoff) $\Delta x \equiv \Lambda^{-1}$ which is sufficiently small in order for the constant and the infinite sum to be negligible, the integral reads

$$\ln\left(\frac{\partial_x}{\Lambda}\right) = -\int_{\Lambda^{-1}}^{\infty} \frac{d\tau}{\tau} \, e^{-\tau \partial_x}$$

or in the more general situation (including the *trace*)

$$\boxed{\mathrm{Tr}\left(\ln\left(\frac{\hat{H}}{\Lambda}\right)\right) = -\int_{\Lambda^{-1}}^{\infty} \frac{d\tau}{\tau} \, \mathrm{Tr}\left(e^{-\tau \hat{H}}\right)}$$



Papers:

- On Gauge Invariance and Vacuum Polarization (1951) - J. Schwinger local pct. 5340
- EIGENTIME IN CLASSICAL AND QUANTUM MECHANICS (1937) - V. A. Fock local pct. 458
- Non-Relativistic Propagators via Schwinger's Method (2007) - A. Aragão, H. Boschi-Filho, C. Farina, F. A. Barone local pct. 7
- Generalized Schwinger Proper Time Method for Dirac Operator with Dynamical Chiral Symmetry Breaking (2002) - Q. Lu, H. Yanga, Q. Wang local pct. 0
- Proper Time Method for Fermions (1998) - A. Das local pct. 0

Documents:

- The DeWitt-Schwinger Proper Time Representation and Heat Kernel (2012) - S.-H. Shao local

Videos:

- Feynman Diagrams in String Theory (2013) - E. Witten

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Schwinger-Keldysh Formalism

The **Schwinger-Keldysh Formalism**, or **Close-time-path Formalism** is a real-time formulation of **finite temperature field theory**.
It uses a closed path in the *complex-time* plane such that the contour goes along the real axis and then back.
From this procedure an effective doubling of the degrees of freedom emerges, such that the **Green functions** are represented by 2×2 matrices.

The technique applies to equlibrium as well as **non-equilibrium systems**.
It has been used for problems in *statistical physics* and **condensed matter theory** such as

- spin systems,
- **superconductivity**,

- lasers,
- **tunneling** and secondary emission,
- plasmas,
- transport processes,
- **symmetry breaking**.

Papers:

- Equilibrium and Nonequilibrium Formalisms Made Unified (1985) - K Chou, Z Su, B Hao, L Yu local pct. 766
- Schwinger-Keldysh Propagators from AdS/CFT Correspondence (2003) - C. P. Herzog, D. Thanh Son local pct. 239

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

String Field Theory

See also:

- **Superstring theory**

Links:

- WIKIPEDIA - String Field Theory
- String Field Theory could be the Foundation of Quantum Mechanics (2014) - R. Perkins

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Thermal Quantum Field Theory

Links:

- WIKIPEDIA - Thermal Quantum Field Theory

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Topological Quantum Field Theory

Papers:

- Topological Quantum Field Theory (1988) - E. Witten local pct. 1825
- On Algebraic Structures Implicit in Topological Quantum Field Theories (1999) - L. Crane, D. Yetter local pct. 73
- Quantum Gravity as Topological Quantum Field Theory (1995) - J. W. Barrett local pct. 47

Links:

- WIKIPEDIA - Topological Quantum Field Theory

Videos:

- Manifold Pairings and Quantum Gravity (2010) - M. Freedman

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Unitary Inequivalence

> ... the analogue of the Stone-von Neumann uniqueness theorem for infinitely many degrees of freedom is false; indeed, in that case, there is an enormously infinite number of unitarily inequivalent representations of the CCR in the Weyl form and, therefore, also of the original CCR. This fact was only slowly and painfully realized, because physicists choose to ignore the restriction in the hypothesis of the Stone-von Neumann uniqueness theorem.
> - Stephen J. Summers -

> Perhaps the single most important problem in the foundations of QFT is the problem of inequivalent representations.
> - David John Baker -

Unitary Inequivalence occurs in systems having an **infinite number of degrees of freedom**.
There are uncountably infinitely many **Unitarily Inequivalent Irreducible Representations** (**URIs**) of the CCRs (see **Heisenberg algebra**) in this case and the choice of proper representation is crucial in any physical application.
It has become clear from rigorous study of concrete models in **constructive quantum field theory** that bosonic systems with identical kinematics but physically distinct dynamics (i.e. when considering forces) require inequivalent representations of the CCRs. Roughly speaking, the kinematical aspects determine the choice of Heisenberg algebra, whereas the dynamics fix the choice of the representation of the given Heisenberg algebra in which to make the relevant, perturbation-free computations. (It is also believed - and proven in a number of indicative special cases - that perturbation series in one representation provide divergent and at best asymptotic approximations to the physically relevant quantities in another, unitarily inequivalent representation).

Different kinds of infinities

A representation of the CCRs can be realized in terms of *creation- and annihilation operators* (satisfying certain (anti-)commutation relations). A (diagonalized) state in this representation is given by

$$|n_0, n_1, \ldots, n_k, \ldots\ldots\rangle$$

where $k \in \mathbb{N}$ and $n_k \in \mathbb{N}$ for bosons and $n_k \in \{0,1\}$ for fermions. k denotes the degrees of freedom alluded to above.
The set of all states will be denoted Γ.

Therefore, the overall number of possible states has **cardinality** $\aleph_0^{\aleph_0} = \aleph_1$ for bosons and $2^{\aleph_0} = \aleph_1$ for fermions.
In any case, the cardinality is that of the **continuum**, $\aleph_1$, i.e. the number of states is uncountably infinite. (See also: **Cantor's diagonal argument**).
Since the number of states is non-denumerable, a **separable Hilbert space** cannot be constructed from Γ and the **Stone-Von Neumann theorem** does not hold. This is where the fundamental difference of **quantum mechanics** and **quantum field theory** lies ! (Actually Q.M. is contained in QFT and the latter is the much broader framework).

Contrary to this, for systems with a finite number of degrees of freedom $k \equiv k_0 < \infty$, the overall number of possible states is $\aleph_0^{k_0} = \aleph_0$ for bosons and even less for fermions. A crucial difference !

Let Γ_0 be the set which contains only a finite number of particles, $\left\{|n_0, n_1, \ldots, n_k, \ldots\ldots\rangle : \sum_{k=0}^{\infty} n_k < \infty\right\}$. (Note, that the number of degrees of freedom is still infinite).
This set of vectors contains the **vacuum** state which has no particles: $|0, 0, 0, \ldots\rangle$ and it spans a Hilbert space in the **Fock space** representation. Its number of basis vectors is countable infinite as the number of degrees of freedom is so.
This is the Hilbert space containing the "bare", "undressed" vacuum. Any other unitarily inequivalent space has an infinite number of particles as seen from this distinguished space.
The reason is this: Applying a finite sequence of creation or annihilation operations to a state will lead to a state that is still within the original Hilbert space.
The application of an infinite sequence of such operations can only annihilate a finite number of already excited states, leaving an infinite number of creation and annihilation operations. If not only a finite number of them are not mutually generating and annihilating particles, one must have an infinite number of particle creations.

On the set $\{\bar{\Gamma}_0\}$ (the complement of $\{\Gamma_0\}$) an equivalence relation can be defined such that each equivalence class $[\bar{\Gamma}_0]$ contains all sequences that differ only in a finite number of places. The set of these equivalence classes $\{[\bar{\Gamma}_0]\}$ is non-denumerable. The vectors corresponding to the sequences in an equivalence class can be used as the basis to construct a Hilbert space. Thus, by defining the creation and annihilation operators on these Hilbert spaces one can build a continuum of UIRs of the CCRs (or CARs = Canonical anticommutation relations) from $\{[\bar{\Gamma}_0]\}$ that are unitarily inequivalent to the Fock representation and among each other.

The non-Fock representations of $\{\bar{\Gamma}_0\}$ are also sometimes called **Myriotic Representations**, describing a quantized field that has creation and annihilation operators satisfying specified commutation rules, but no vacuum state.

Another way to explain the reason that there are an uncountable number of UIRs is that there are an uncountable number of ways of choosing a countable subset from an uncountable set.

Not surprisingly, unitary inequivalence has a deep implications for the **philosophy of physics**, in particular that of quantum field theory.

Flat spacetime

There is a distinguished Hilbert space, namely the one which contains the zero particle state. Does this correspond with a flat spacetime ? What speaks for this is that it is the Hilbert space that high energy physicists like to use, who usually don't care about **gravity** and curvature. One uses this space for the "in-" and "out-states" assuming that the incoming and outgoing particles come from and go to a Minkowski vacuum. (Yet given the known global geometry of spacetime, this can at best be a very good approximation - which in fact it is, as is demonstrated by innumerous (scattering) experiments in high energy physics). The states involving the interactions are encoded by the **S-matrix**. Due to **Haag's theorem** these must "live" in another Hilbert space which presumably then is unitarily inequivalent to the one of the in- and out states. That is to say that one could think of forces, bringing in the dynamics, as introducing as key element, non-unitarity, resulting in virtual particles, "dressed" physical values, infinities, etc. This situation can be brought under control by parameterizing the *coupling constants* and carrying out **renormalization**. Does this mean that following the **renormalization group** flow means "running" through unitarily inequivalent Hilbert spaces (= "running of the coupling constants") ?
Also, in this scenario there seems to be no hope for constructing a theory of **quantum gravity** in a single Hilbert space. (Interestingly it has been shown (Stelle, 1977) that gravity in fact is renormalizable, if one dispenses with unitarity).
To be consequent, one had to include gravity in the S-matrix, but then the usual procedure doesn't go through because there are no free in- and out states any more. Rather, the whole universe had to serve as the object to be scattered at - quite of an oddity though. (Here it may be good advice to ask condensed matter physicists, who face similar situations in the laboratory, e.g. **phase transitions**).
Another picture that arises is that in the conventional approach the in- and out states at "unitary infinity", which are the ones that are measured, are **"collapsed" states** which correspond with particles, whereas the states in between, described by the S-matrix, are virtual particles, those are the particles involving forces/dynamics, etc.
An interesting question that arises is this: It seems that short range forces are less problematic than long range ones, as if one goes far enough away from the spot of interactions the former are negligible for all practical purposes. This is why gravity may pose a problem. But then, why is quantum electrodynamics so successful ? (Yet, in fact, it is known that there are also problems with this theory in very high orders, where presumably it breaks down).

Some further thoughts

As QFT is based on the continuum whereas an ordinary computer (also a **quantum computer**, as it can be mapped to a Turing machine) is based on a sequence of no more than a countably infinite number of calculation steps (thereby facing the **halting problem**), the issue of **noncomputability** of the **conscious brain** (e.g. advocated by **Roger Penrose**) comes to mind.
If consciousness really encompasses unitarily inequivalent vacua, then it would easily outperform any Turing machine. (In fact modelling the brain by means of QFT seems to be feasible - see **quantum brain dynamics**).
If this were so, to achieve true **AI** one had to harness QFT.
This would also imply that **quantum consciousness**, merely based on quantum mechanics, does not work. It therefore may be interesting to think about how to built a computer based on QFT, a **quantum field computer**.

Unitary inequivalence may also be related to a **gravitationally induced state reduction** in the context of consciousness (e.g. "**Orch-OR reduction**").

... to be continued ...

Papers:

- Representations of the Anticommutation Relations (1954) - L. Gårding, A. Wightman local pct. 101
- Representations of the Commutation Relations (1954) - L. Gårding, A. Wightman local pct. 83
- Clifford Geometric Parameterization of Inequivalent Vacua (1997) - B. Fauser local pct. 21

- Explaining Quantum Spontaneous Symmetry Breaking (2004) - C. Liu, G. G. Emch local pct. 16 prl. 10
- Unitarily Inequivalent Representations in Algebraic Quantum Theory (2005) - FM Kronz, T. A Lupher local pct. 9
- Goldstone Theorem, Hugenholtz-Pines Theorem and Ward-Takahashi Relation in Finite Volume Bose-Einstein Condensed Gases (2005) - H. Enomoto, M. Okumura, Y. Yamanaka local pct. 6
- On Representations of Finite Type (1998) - R. V. Kadison local pct. 1
- How to Construct Unitarily Inequivalent Representations in Quantum Field Theory - T. Lupher local pct. 0
- Quantum Phase Transition, Dissipation, and Measurement (2009) - S. Chakravarty local pct. 0

Theses:

- The Philosophical Significance of Unitarily Inequivalent Representations in Quantum Field Theory (2008) - T. A. Lupher local tct. 2 trl. 10
- Quantum Field Theory and Phase Transitions - Symmetry Breaking and Unitary Inequivalence (2010) - D. Sánchez de la Peña local

Links:

- Website of Tracy Lupher

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Vacuum Entanglement

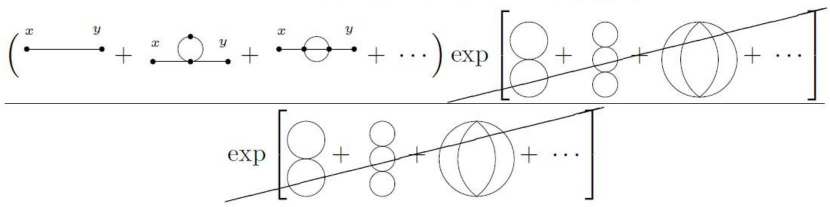
See **Reeh-Schlieder Theorem**.

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Vacuum Fluctuations

In **quantum field theory** **Vacuum Fluctuations** correspond with vacuum diagrams (a.k.a. "bubble diagrams", i.e. *Feynman diagrams* without external lines).
BUT these diagrams appear in the numerator AND the denominator of the *Gell-Mann–Low formula* and cancel each other,

$$\langle 0 | \mathrm{T}\left[\Phi(x)\Phi(y)\right] | 0 \rangle = \frac{\langle 0 | \mathrm{T}\left[\phi(x)\phi(y)\exp(-i\int dt H_{int}(t))\right] | 0 \rangle}{\langle 0 | \mathrm{T}\left[\exp(-i\int dt H_{int}(t)))\right] | 0 \rangle} =$$



The overall contribution of these diagrams is infinite.

It thus seems doubtful to say that the **vacuum** is in a state of constant activity, with the spontaneous creation and annihilation of virtual particles, because any such "particle" does not interact with the observable matter of the universe, and therefore should not be regarded as a part of physical reality. In other words, vacuum fluctuations are not experimentally accessible.

However, the situation may be different if one takes into account **gravity**.

Papers:

- Vacuum Fluctuations of Energy Density can Lead to the Observed Cosmological Constant (2004) - T. Padmanabhan local pct. 102
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Your comments are very much appreciated. Suggestions, questions, critique, ... ?

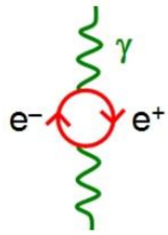
Vacuum Polarization

Vacuum Polarization is an effect in **quantum field theory** which results from the **uncertainty principle**, giving rise to the spontaneous creation of *virtual particles* (charged particle-anti particle pairs).

An important consequence of the polarization of the vacuum is that it effectively reduces the charge of a particle (a.k.a. **Screening**). The reduction is dependent on distance and hence on the energy scale. The effect is larger at shorter distances (i.e. higher energies). It leads to a running of the *coupling constant* associated with the charge.

QED

The contribution in second order **perturbation theory** is given by the following kind of *Feynman diagram*:



Vacuum polarization has been experimentally verified in the context of the following effects:

- Lamb shift
- **Anomalous magnetic moment**

Papers:

- On Gauge Invariance and Vacuum Polarization (1951) - J. Schwinger local pct. 5293

Lectures:

- Lectures on the Physics of Vacuum Polarization: from GeV to TeV Scale (2009) - F. Jegerlehner local

Links:

- WIKIPEDIA - Vacuum Polarization

Videos:

- Photon Self Energy I - P. Tripathy

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Wavefunction Renormalization

Links:

- WIKIPEDIA - Wavefunction Renormalization

Your comments are very much appreciated. Suggestions, questions, critique, ... ?

Wick's Theorem

Wick's Theorem establishes the relationship between the *time ordered product* of n fields and a sum of **normal ordered products**. In case of scalar fields, this may be expressed for n even as

$$\begin{aligned}\mathcal{T}\left[\phi(x_1)\cdots\phi(x_n)\right] = \; & :\phi(x_1)\cdots\phi(x_n): + \\ & \sum_{\text{perm}} \langle 0|\mathcal{T}\phi(x_1)\phi(x_2)|0\rangle :\phi(x_3)\cdots\phi(x_n): + \\ & \sum_{\text{perm}} \langle 0|\mathcal{T}\phi(x_1)\phi(x_2)|0\rangle\langle 0|\mathcal{T}\phi(x_3)\phi(x_4)|0\rangle :\phi(x_5)\cdots\phi(x_n): + \\ & \vdots \\ & \sum_{\text{perm}} \langle 0|\mathcal{T}\phi(x_1)\phi(x_2)|0\rangle\cdots\langle 0|\mathcal{T}\phi(x_{n-1})\phi(x_n)|0\rangle\end{aligned}$$

where the summation is over all the distinct ways in which one may pair up fields.
The result for n odd looks the same except for the last line which reads

$$\sum_{\text{perm}} \langle 0|\mathcal{T}\phi(x_1)\phi(x_2)|0\rangle\cdots\langle 0|\mathcal{T}\phi(x_{n-2})\phi(x_{n-1})|0\rangle :\phi(x_n):$$

Expressions of the type $\langle 0|\mathcal{T}\phi(x_i)\phi(x_j)|0\rangle$ are c-numbers which are also called **Contractions** in this context for which alternative notations are used such as dots or lines connecting the respective fields.

Wick's theorem allows for expressing the **Dyson series expansion** (or **S-matrix**) in terms of normal ordered products and *Feynman propagators* instead of time ordered products, simplifying computations.

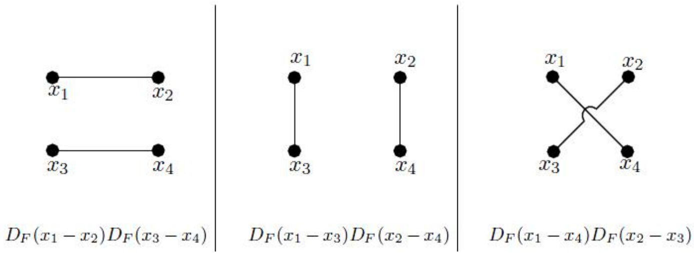
Contractions are equivalent to Feynman propagators, i.e.

$$\langle 0|\mathcal{T}\phi(x_1)\phi(x_2)|0\rangle = D_F(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4}\,\frac{e^{-ik(x_1-x_2)}}{(k^2-m^2)+i\varepsilon}$$

Examples

A pictorial representations of contractions in terms of Feynman diagrams looks as follows:

$$\begin{aligned}\langle 0|\,T\{\phi_1\phi_2\phi_3\phi_4\}\,|0\rangle = & D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3)D_F(x_2-x_4) \\ & + D_F(x_1-x_4)D_F(x_2-x_3)\end{aligned}$$



Papers:

- The Evaluation of the Collision Matrix (1950) - G. C. Wick local pct. 921
- On the Hopf Algebraic Origin of Wick Normal-ordering (2000) - B. Fauser local pct. 54

Links:

- WIKIPEDIA - Wick's Theorem

Videos:

- Interacting Field Theory - III - P. Tripathy
- Quantum Field Theory II - Lecture 3 (2011) - F. David - 54:00-
- Lectures on Quantum Field Theory (2009) - D. Tong - Lecture 8

Your comments are very much appreciated. Suggestions, questions, critique, ... ?